A HYBRID GENETIC ALGORITHM FOR VRPSTW USING COLUMN GENERATION

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Introduction

City Logistics **deals with the measure to alleviate truck based Urban Freight related problems.**

Problems Such as

Accidents

On street parking

Environmental Problems like generation of NOx, SPM and CO_2, etc.

Measures such as

Route Optimization

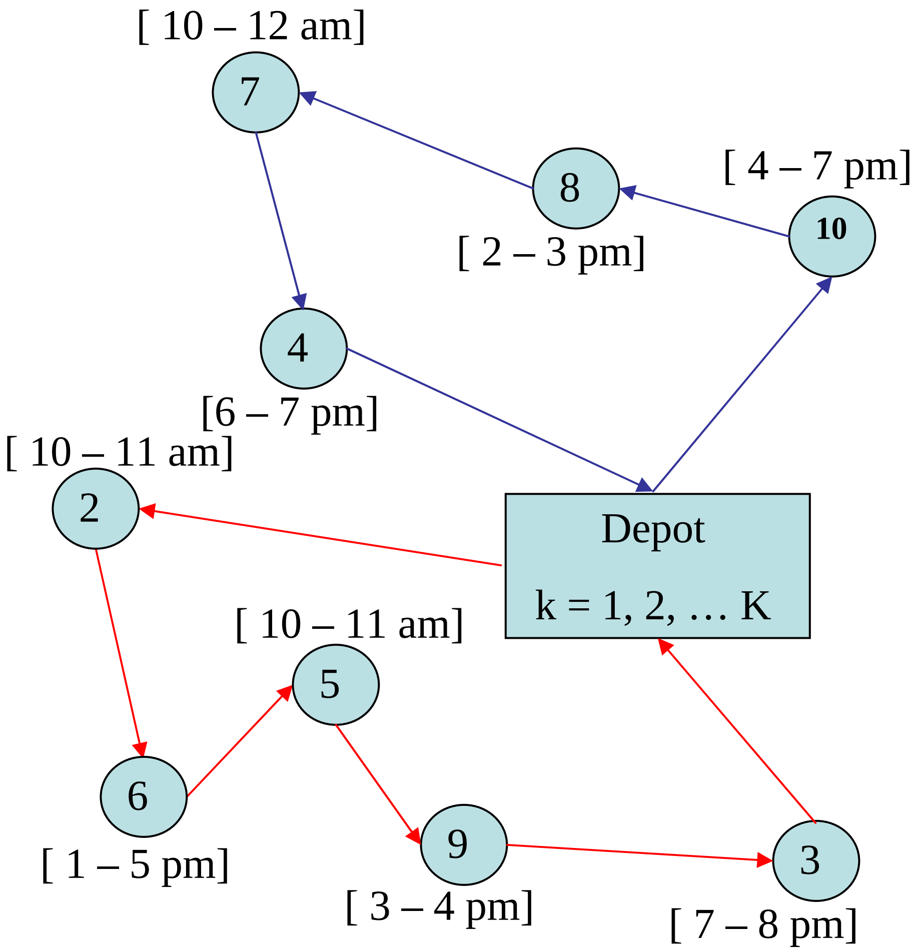
Ideal location of Logistic Terminals

Controlling Load Factors

etc.

Vehicle Routing and scheduling Problem (VRP) with its various variants is used to evaluate many such measures.

VRPTW



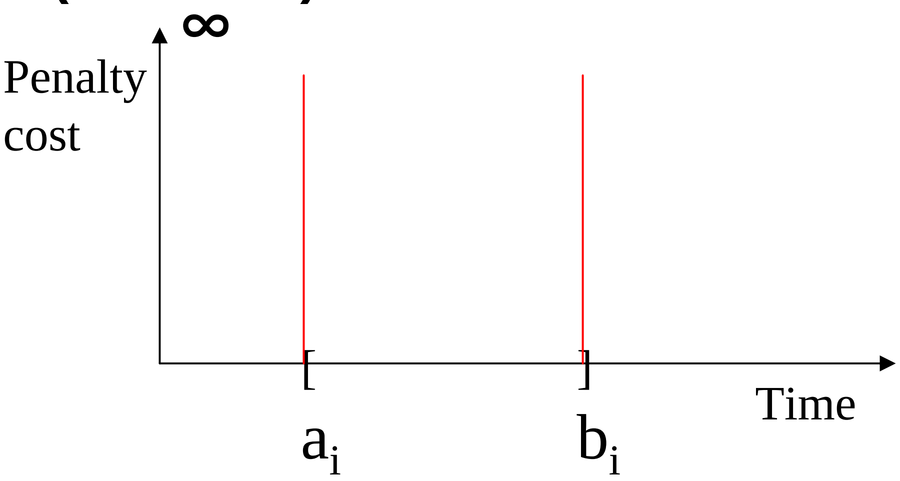
Vehicle Routing Problem with Time Windows (VRPTW) is defined as to find the minimum cost routes for *k* vehicles to service all the clients.

Constraint: A vehicle can not serve more clients than its capacity. delivery at each client must be within some pre-defined time windows.

VRPTW Variants

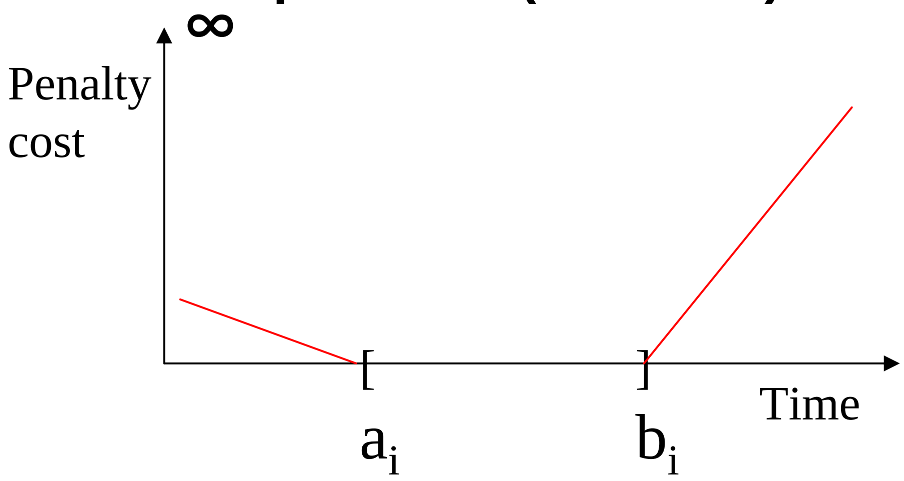
Hard Time Windows

Delivery is not possible outside the specified Time Windows (VRPHTW).



Soft Time Windows

Delivery is possible outside the specified Time Windows with penalties (VRPSTW).



Solution Approaches

Most of the exact optimization research (e.g. column generation, lagrangian relaxation) is directed towards VRPHTW, while heuristic approaches (e.g. Tabu search, Genetic Algorithms) are more common for VRPSTW.

Objectives

Exact solution approach for VRPSTW is always desirable.

Column generation offers a flexible framework to start with.

Column Generation Master Problem

Subproblem

(column generator)

Complex soft time window constraints and time dependent costs make it difficult to formulate an exact subproblem.

Compromise

A Hybrid Approach

Column Generation Master Problem

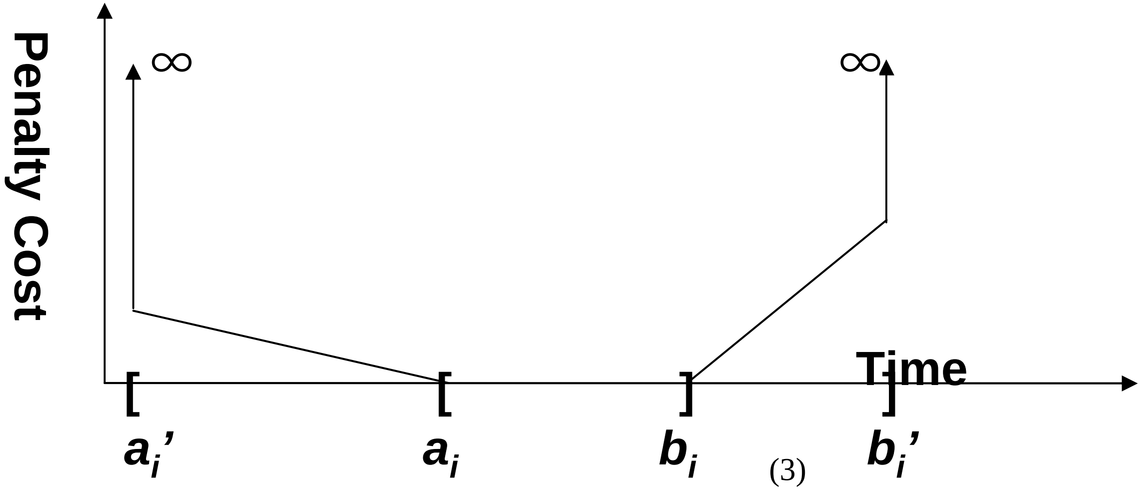
Heuristic Subproblem

(column generator)

Compared to heuristic

- Solution quality
- Speed
- Consistency

Relaxing Time Windows



(3)

$$a_i' = \max\left[0, a_i - \frac{(c_{0i} + c_{i0})}{c_e}\right] \qquad b_i' = \min\left[b_0 - t_{i0}, \quad b_i + \frac{(c_{0i} + c_{i0})}{c_l}\right]$$

$$c'_{ij} = \begin{cases} c_{ij}, & \text{if } a_j \le s_{jk}' \le b_j \\ c_{ij} + c_l\,(s_{jk}' - b_j) & \text{if } b_j < s_{jk}' \le b_j' \\ c_{ij} + c_e\,(s_{jk}' - a_j) & \text{if } a_j' \le s_{jk}' < a_j \end{cases}$$

VRPTW Formulation

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c'_{ij} x_{ijk} \qquad (1)$$

subject to

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} = 1 \qquad \forall\, i \in C \qquad (2)$$

$$\sum_{i \in C} d_i \sum_{j \in V} x_{ijk} \le q \qquad \forall\, k \in K \qquad (3)$$

$$\sum_{j \in V} x_{0jk} = 1 \qquad \forall\, k \in K \qquad (4)$$

$$\sum_{i \in V} x_{ihk} - \sum_{j \in V} x_{hjk} = 0 \qquad \forall\, h \in C, \qquad \forall\, k \in K \qquad (5)$$

$$\sum_{i \in V} x_{i0k} = 1 \qquad \forall\, k \in K \qquad (6)$$

$$a'_i \le s'_{ik} \le b'_i \qquad \forall\, i \in V, \qquad \forall\, k \in K \qquad (7)$$

$$a_i \le s_{ik} \le b'_i \qquad \forall\, i \in V, \qquad \forall\, k \in K \qquad (8)$$

$$s_{ik} + t_{ij} - s_{jk} \le (1 - x_{ijk}) M_{ijk} \qquad \forall\, (i, j) \in A, \qquad \forall\, k \in K \qquad (9)$$

$$x_{ijk} \in \{0, 1\} \qquad \forall\, (i, j) \in A, \qquad \forall\, k \in K \qquad (10)$$

Hybrid Genetic Algorithm with Column Generation Heuristic (HGACGH)

Column generation or Dantzig-Wolfe decomposition, decomposes the VRPTW problem into:

*** (Sub-Problem 3 – 10). Provides feasible routes (col.) of negative reduce cost.**

*** (Master Problem) Set partitioning problem, assigns best routes to vehicles.**

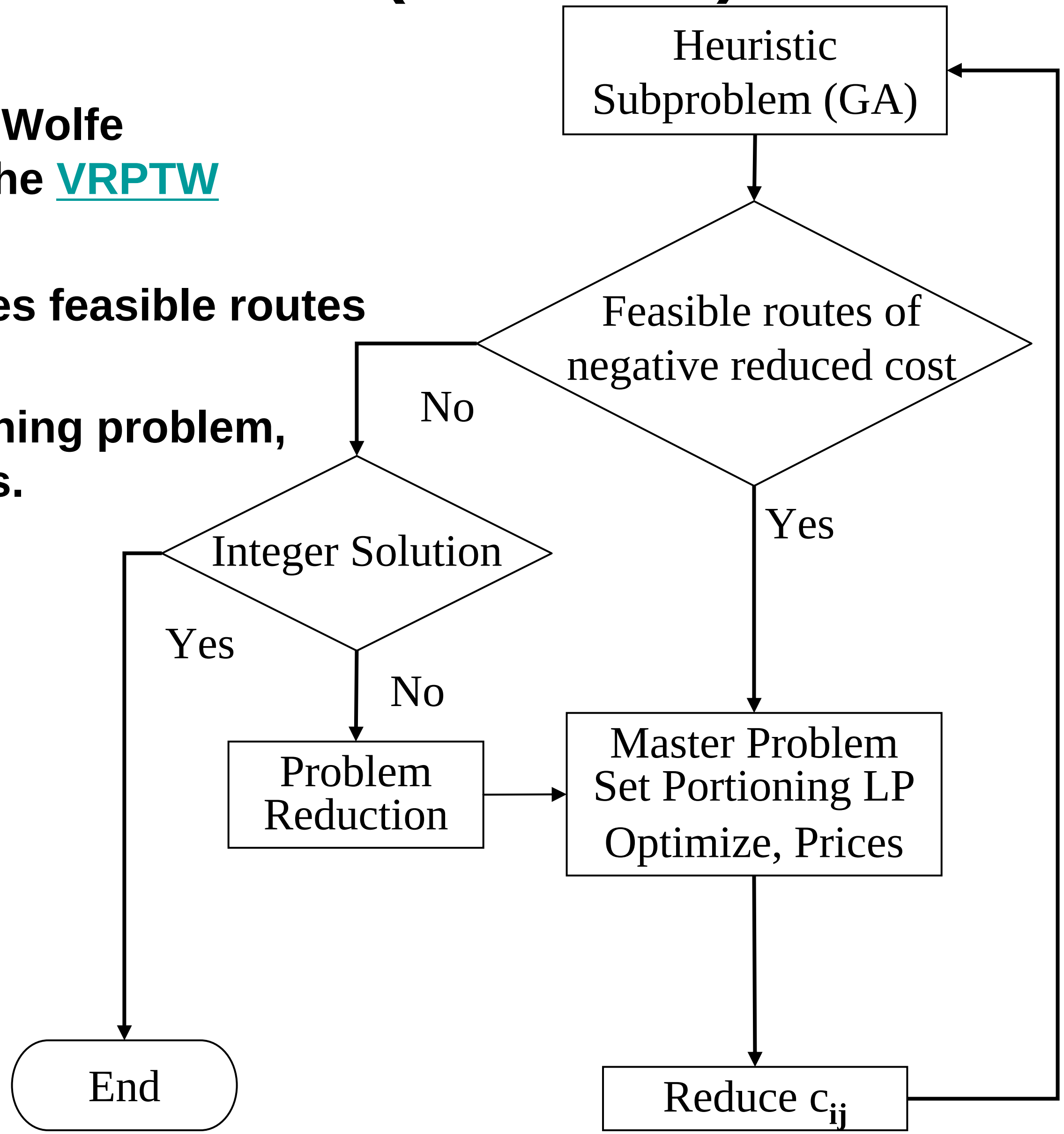
$$\min \sum_{p \in P} c_p y_p$$

subject to

$$\sum_{p \in P} a_{ip} y_p = 1 \qquad \forall i \in C$$

$$y_p \in \{0,1\} \qquad \forall p \in P$$

Feillet et al., 2004.



Heuristic Subproblem: Genetic Algorithms

Objective: Minimize the total cost including running and penalty costs. Running cost depends on the reduced cost matrix: $\overline{c'}_{ij}$

No. of Generations: 30

Initialization: Stochastic Push Forward Insertion Heuristic (SPFIH)

(Alvarenga and Mateus, 2004)

Population Size: 500, integer value genes, each chromosome represents a feasible solution with extra information such as route starting nodes.

Crossover: Order based crossover, to prevent structure of gene
Crossover Rate = 98 %

Mutation: Single Customer Route Elimination Mutation (SCREM)
Mutation Rate = 4%

Integer Solution

If the solution is not integer at the end of the Column Generation algorithm due to any one of its stopping criterions

- **Number of Column Generation iterations**
- **No change in the prices**
- **No negative reduce cost columns from heuristic subproblem**

Integer solution were obtained by successive problem reductions by

- **Extracting routes based on the columns generated so far covering at least half of the customers**
- **Avoiding single customer routes as many as possible**
- **Formulating a smaller VRPSTW problem for the remaining customers**

Heuristic Solution Technique: Genetic Algorithms

No. of Generations: 250 x (No. of Customers)

Initialization: Stochastic Push Forward Insertion Heuristic (SPFIH)

(Alvarenga and Mateus, 2004)

Population Size: 500, integer value genes, refreshed after every 500 generations keeping 4% of present and generating remaining 96% with SPFIH.

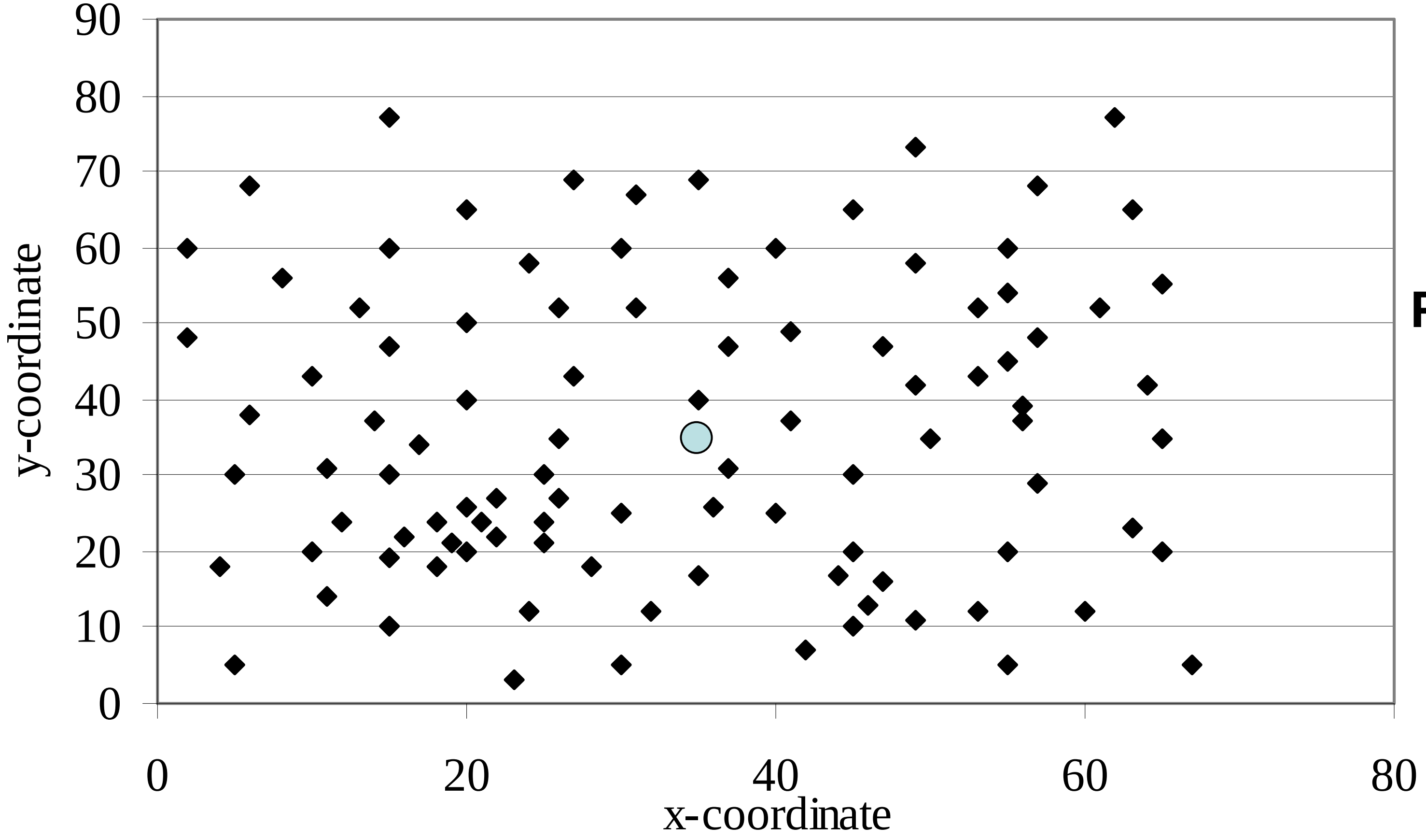
Crossover: Order based crossover, to prevent structure of gene
Crossover Rate = 98 %

Mutation: Swap Mutation, to prevent structure of gene
Mutation Rate = 5%

Test Instances

Solomon's R1 type benchmark instances

- **R101-100** **R101-50**
- **R102-100** **R102-50**
- **R103-100** **R103-50**



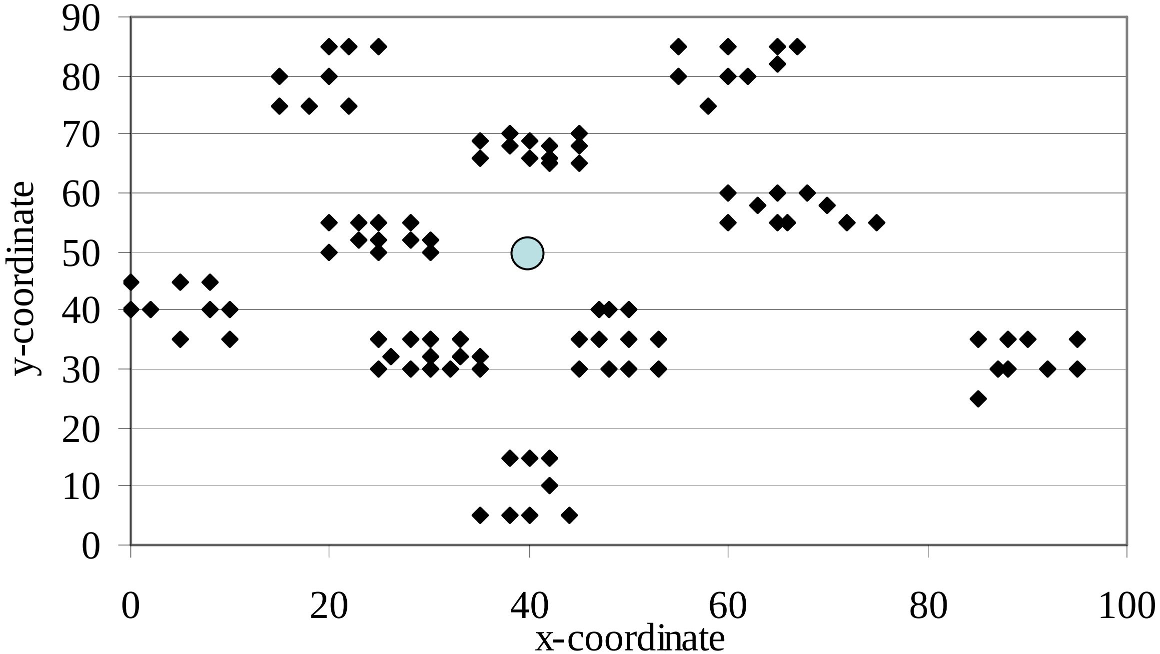
R101-100

Validation

Solomon's C101-100 benchmark instance

* Customers located into 10 Clusters
* Time windows of clustered customers matched
* VRPHTW best solution 827.3 with 10 vehicles, no waiting
* Vehicles bound by the customer demands

Best solution found by HGACGH, 827.3, 10 Vehicles (best integer solution before any problem reduction step)



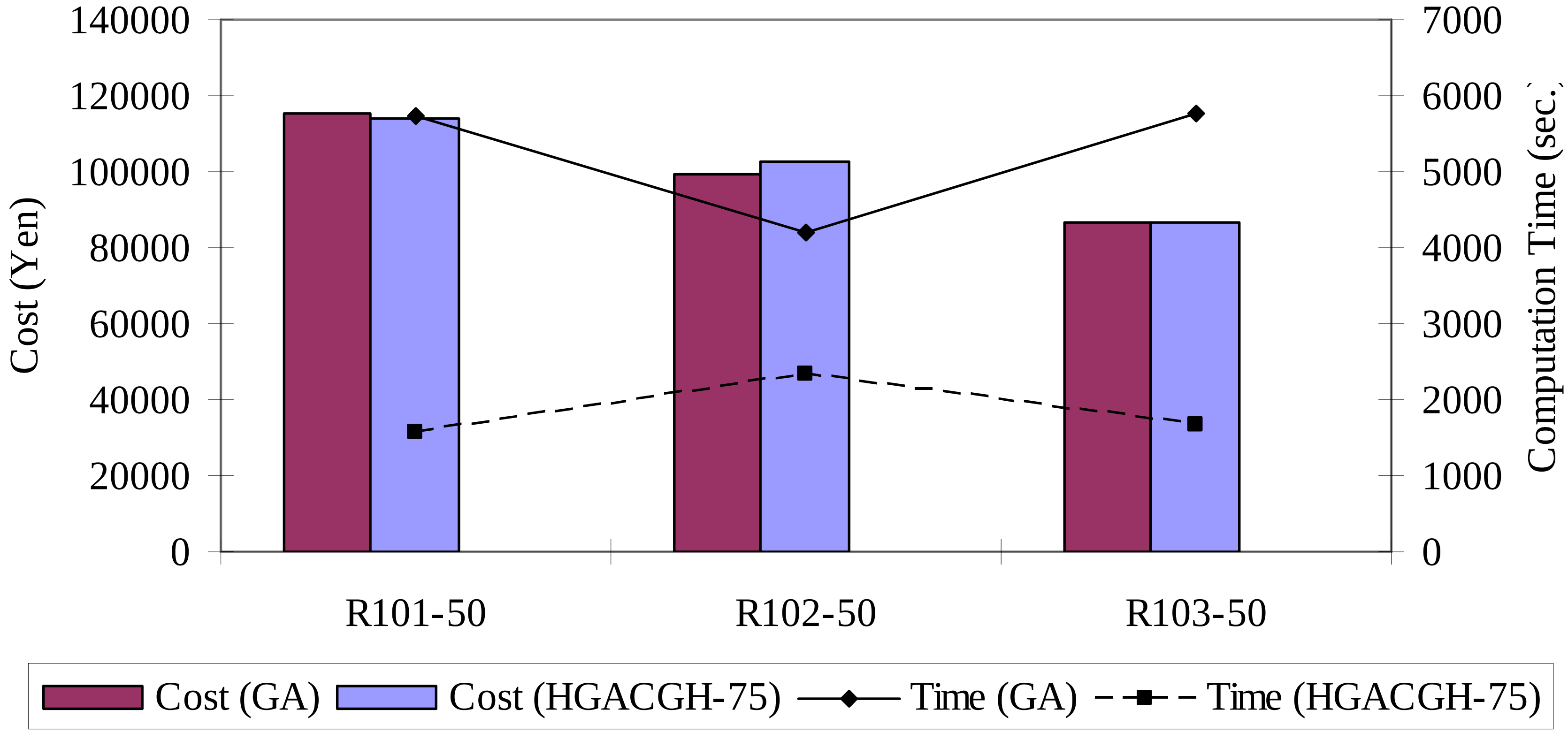
C101-100

Results of HGACGH

50 customers instance

* Solution quality is almost same as GA

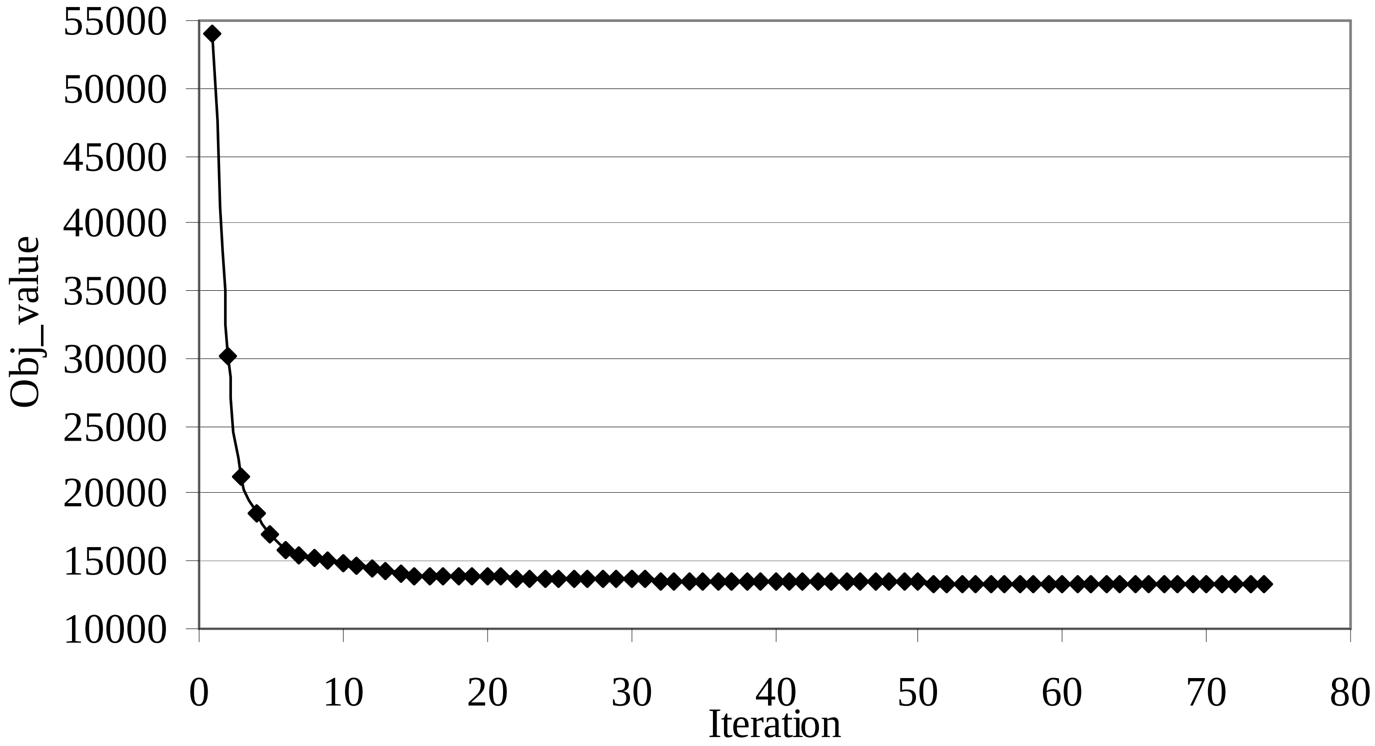
* Computation time about 1/3rd of GA



Best solution comparison by HGACGH, and GA for 50 customer test instances

Results of HGACGH

* Initially 75 column generation iterations were set
* Relative decrease in objective value decrease with iterations
* Algorithm was also tried with 50 – column generation iterations

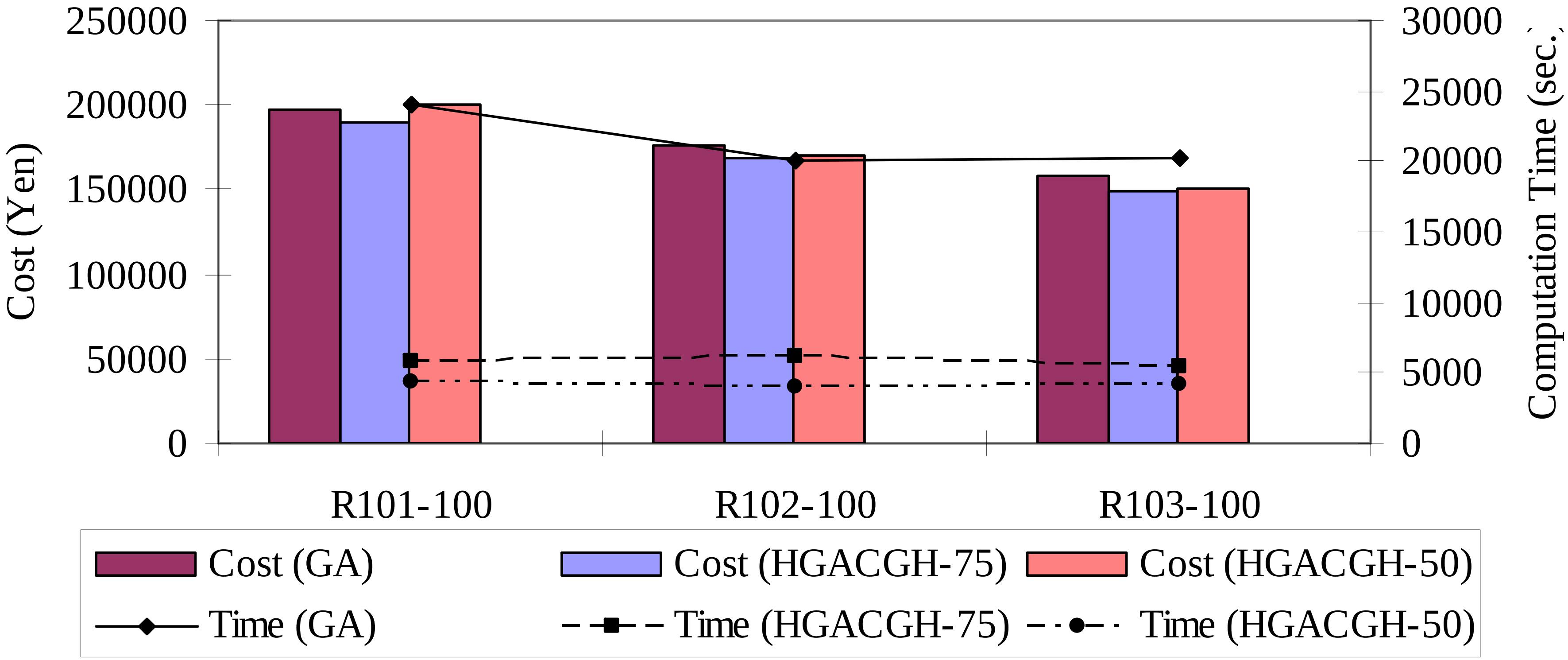


Objective value convergence in R101-100 (Run 1) in HGACGH before problem reduction

Results of HGACGH

100 customer instances

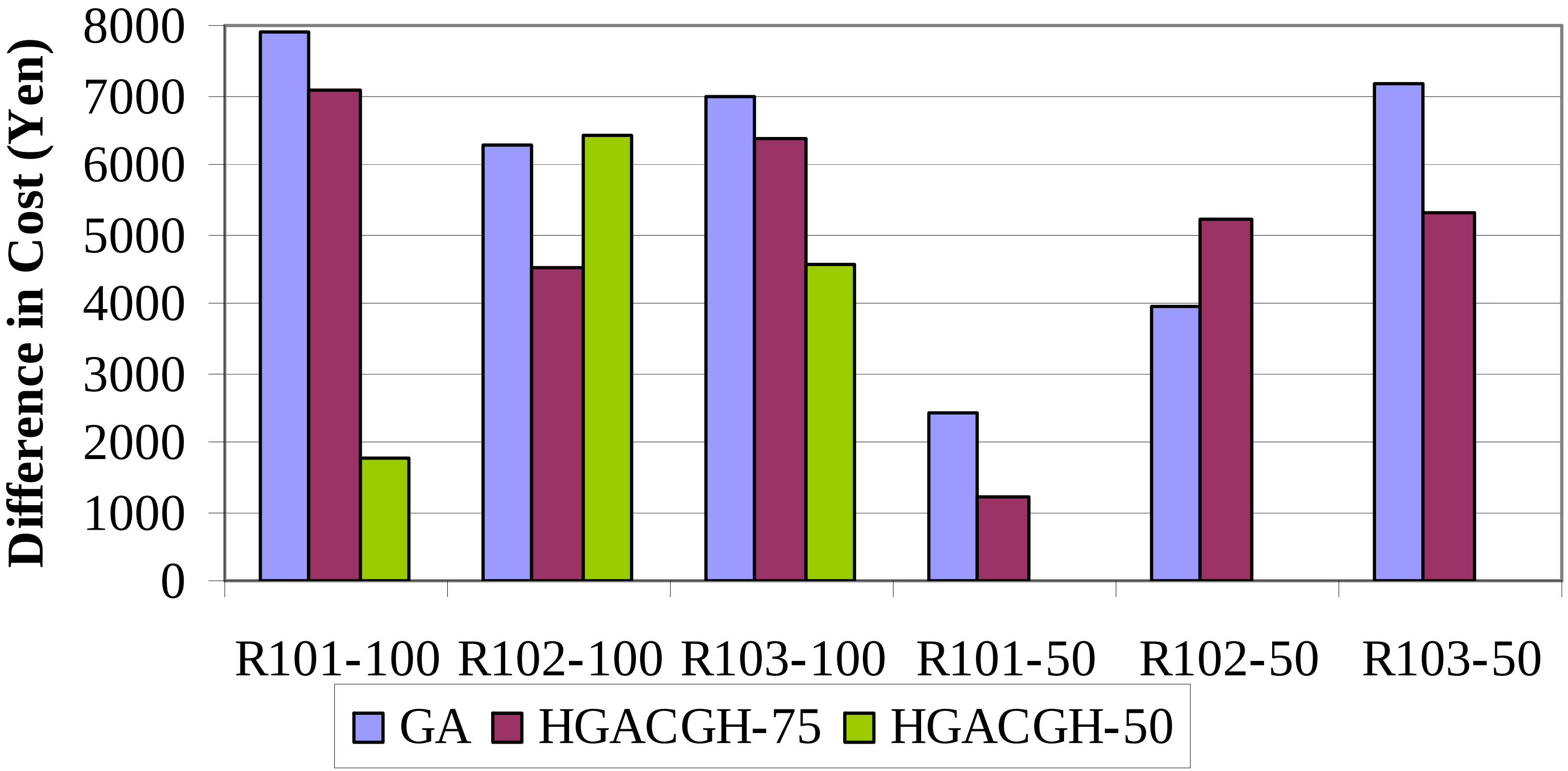
- * **Better solution quality**
- * **Remarkably lower computation times**



Best solution comparison by HGACGH, and GA for 100 customer test instances

Results of HGACGH

Three runs were made for each test instance for GA and HGACGH, consistency was observed by relative difference in the best and average solutions.



Cost difference comparisons between average and best solutions

Conclusions

• Column generation schemes provide flexible framework to develop better heuristics for VRPSTW despite its complex time windows constraints and time dependent costs.

• Using the dual information (shadow prices) from master problem, heuristic subproblem was able to provide negative reduced cost columns of sufficient quality.

• This resulted in

* Rapid decrease in objective value and over all better solution quality as compared to simple GA.

* Less computation time

* More consistence hybrid heuristic

Thank you for your attention

Your Questions and Comments are welcome