A Conceptual Framework for Solving the Multiple Depot Probabilistic Vehicle Routing Problem with Time Windows (MDPVRPTW)

by

Sutapa Samanta

(s_sutapa12@yahoo.com)

Doctoral student,

Morgan State University

and

Manoj K. Jha

(mkjha@eng.morgan.edu)

Assistant Professor,

Morgan State University

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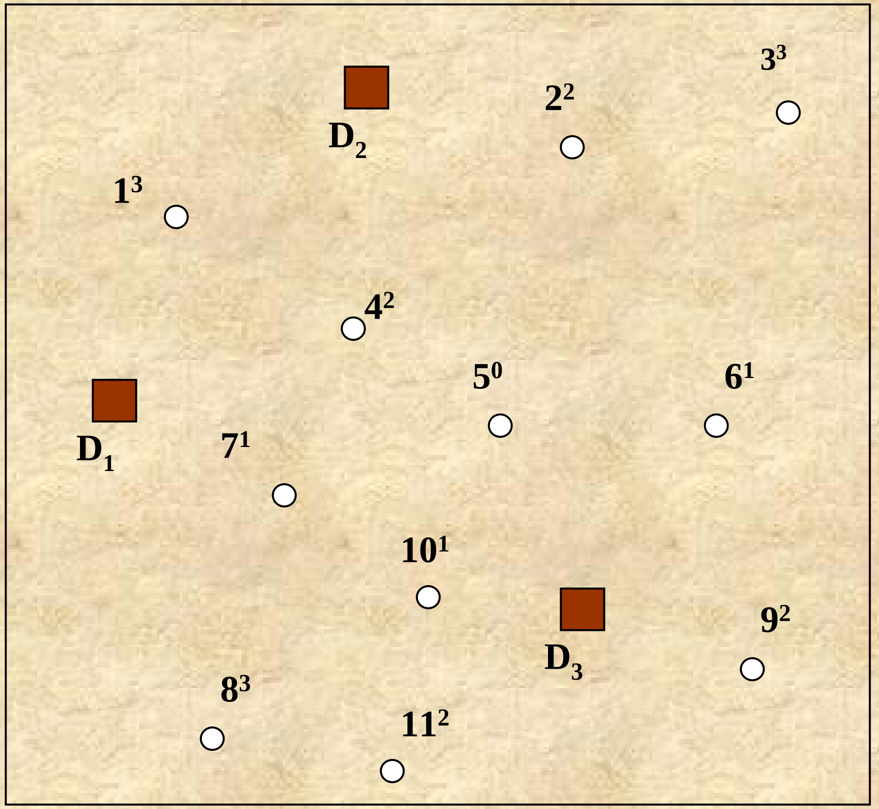
The problem definition

- Vehicle Routing Problem
- Multiple Depots
- Probabilistic Demand
- Time Windows



Earlier works done:

- Vehicle routing problem (VRP) (Golden, 1984)
- Vehicle routing problem with time window (Kohl and Madsen, 1997)
- Vehicle routing problem with stochastic demand (Jailet and Odoni, 1988, Bertsimas et al., 1991)

- Multi depot Vehicle routing problem (Nagy and Salhi, 2005)

- Multi depot Vehicle routing problem with stochastic demand (Chan et al., 2001)

- Vehicle routing problem with time windows and stochastic demand (Barmel and Simchi-Levi, 1996)

Proposed Methodology

- Formulating the problem as a mixed integer programming problem
- Searching for the optimum solution using Genetic Algorithm

Problem formulation:

Minimize

$$Z = \sum_{k \in M} \sum_{i \in N \cup K} \sum_{j \in N \cup K} C_{ij} \cdot d_{ij} \cdot x_{ijk} + C_w \times \sum_i w_i + C_d \times \sum_i d_i$$

- The objective function is to minimize the total travel cost embedded with the penalty for the delay and waiting at the nodes

Subject to

The constraints (given by equations 1-9)

Notations used:

- $N = \{1,2,\ldots.n\}$ set of customers
- $K = \{1,2, \ldots.k\}$ set of depots
- $x_{ijk} = \begin{cases} 1, & \text{if arc } ij \text{ is part of route } k \\ 0, & \text{otherwise} \end{cases}$
- $M = \{1,2,\ldots.m\}$ set of vehicles
- d_{ij} = Distance between customer i and j
- c_{ij} = Cost per unit travel distance
- C_w = Penalty for the early arriving at the customer node i
- C_d = Penalty for late arriving at the customer node i
- (a_i, b_i) = Time window for customer node i
- t_i = Actual arrival time at node i
- w_i = Waiting time at the customer node i
- d_i = Delay time at the customer node i
- D_{max} = Maximum distance which the vehicles may cover in a tour
- q_i = Demand of customer i
- Qm = Capacity of vehicle m
- $q(m)$ = Total demand picked up by vehicle m
- p_i = Probability value of the demand at node i

Constraints

$$q(m) \leq Q_m \qquad m \in M \qquad \text{....(1)}$$

$$q(m) = \sum_{i \in N} p_i q_i \qquad \text{....(2)}$$

Constraints (1) : maximum capacity of the vehicle

Equation (2) : the total demand which consists of probabilistic values of the customer demand

$$\sum_{i \in N \cup K} \sum_{j \in N \cup K} d_{ij} \cdot x_{ijk} \leq D_{\max} \qquad k \in M \qquad \text{....(3)}$$

Constraint (3) : maximum distance a vehicle can travel

Continued..

$$\sum_{k\in M}\sum_{i\in N\cup K} x_{ijk} = 1 \qquad j\in N \qquad \text{....(4)}$$

$$\sum_{k\in M}\sum_{i\in N\cup K} x_{ijk} \geq 1 \qquad j\in K \qquad \text{....(5)}$$

Constraints (4) and (5) : every customer belongs to one and only one route or vehicle

$$\sum_{k\in M}\sum_{i\in N\cup K} x_{ijk} \leq 1 \qquad k\in M \qquad \text{....(6)}$$

Constraint (6) : a vehicle can depart only once from a node

$$\sum_{i\in N\cup K} x_{ijk} = \sum_{i\in N\cup K} x_{jik} \qquad j\in N\cup K, k\in M \qquad \text{....(7)}$$

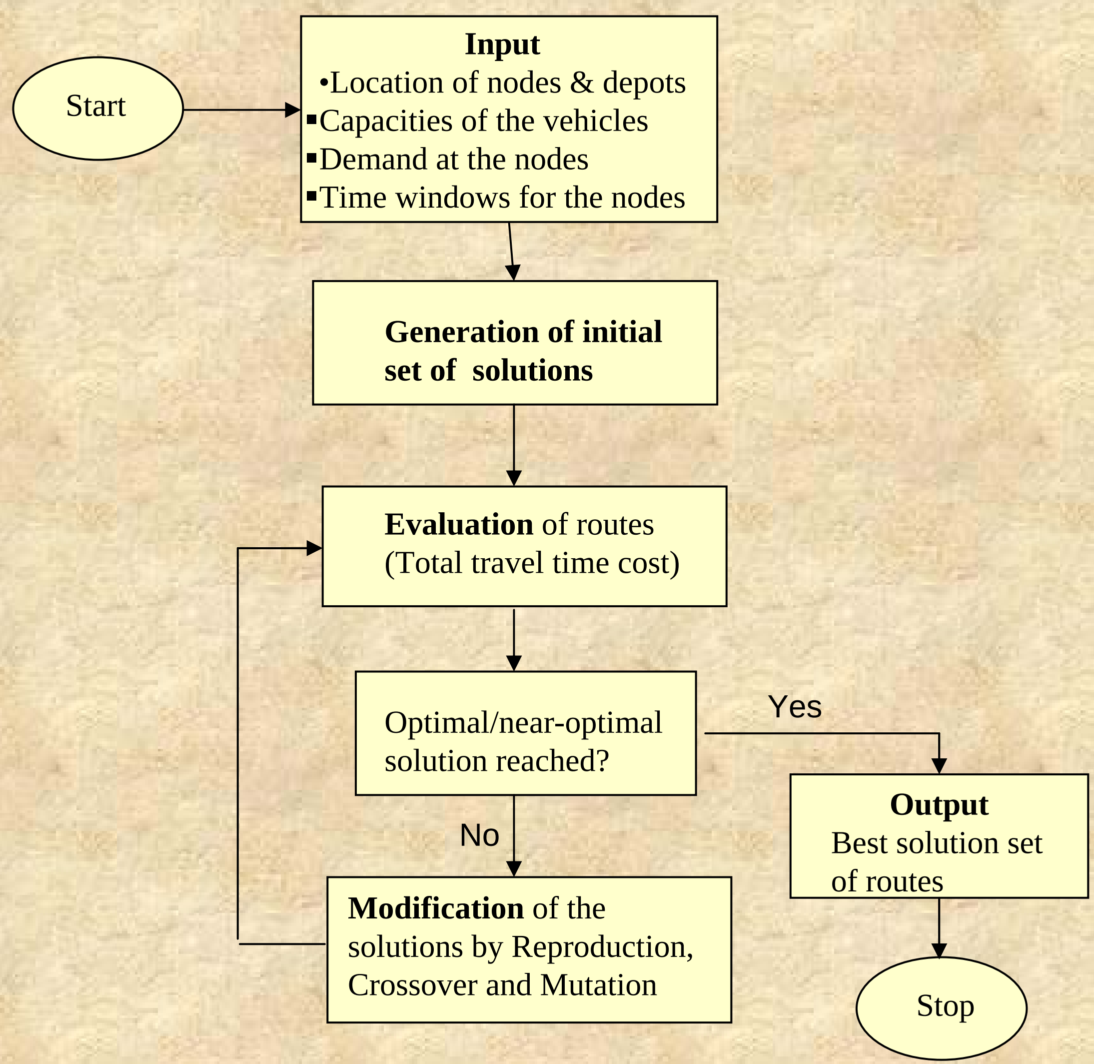
Constraint (7) : every customer is entered and left by the same vehicle

$$w_i \geq a_i - t_i \qquad i\in N \qquad \text{....(8)}$$

$$d_i \geq t_i - b_i \qquad i\in N \qquad \text{....(9)}$$

Constraints (8) and (9) : the time window constraints

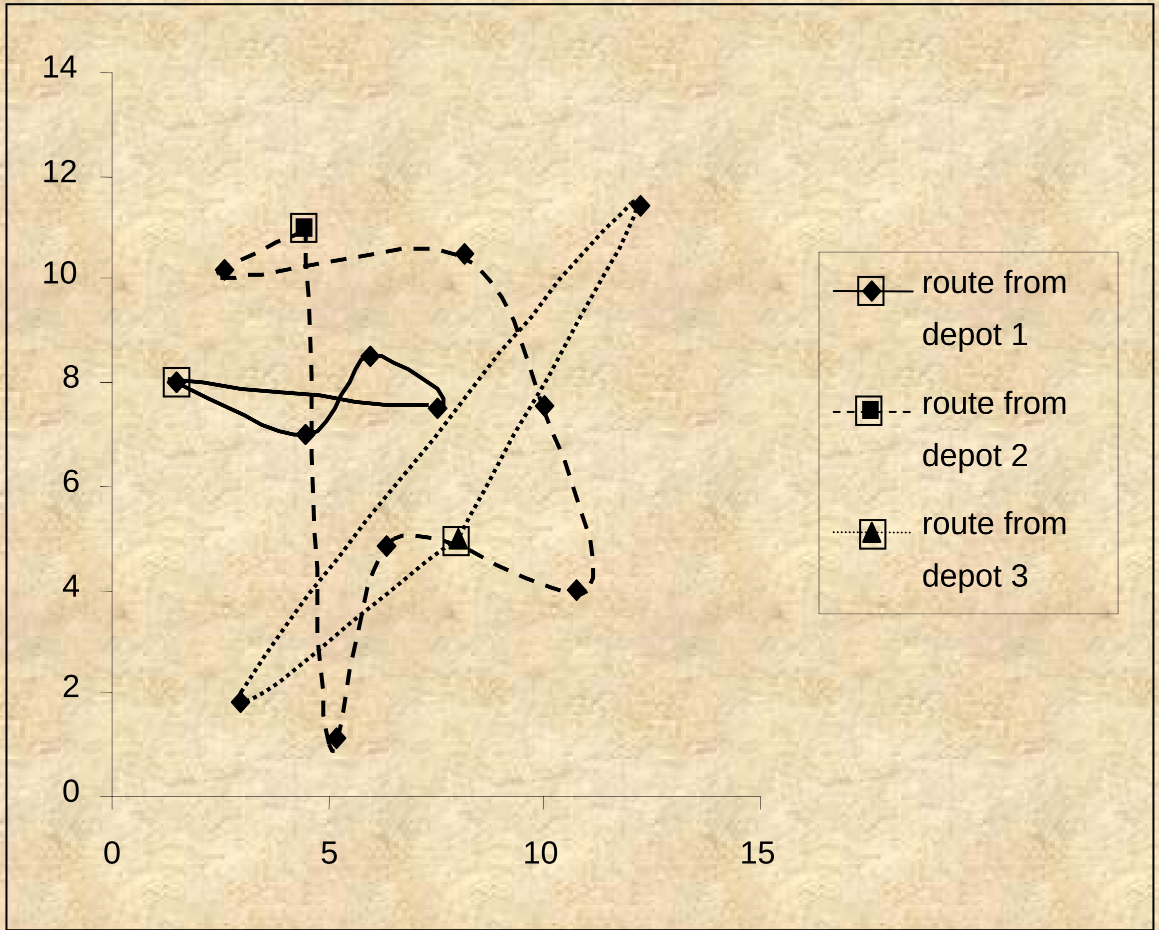
Application of Genetic Algorithm: Flowchart



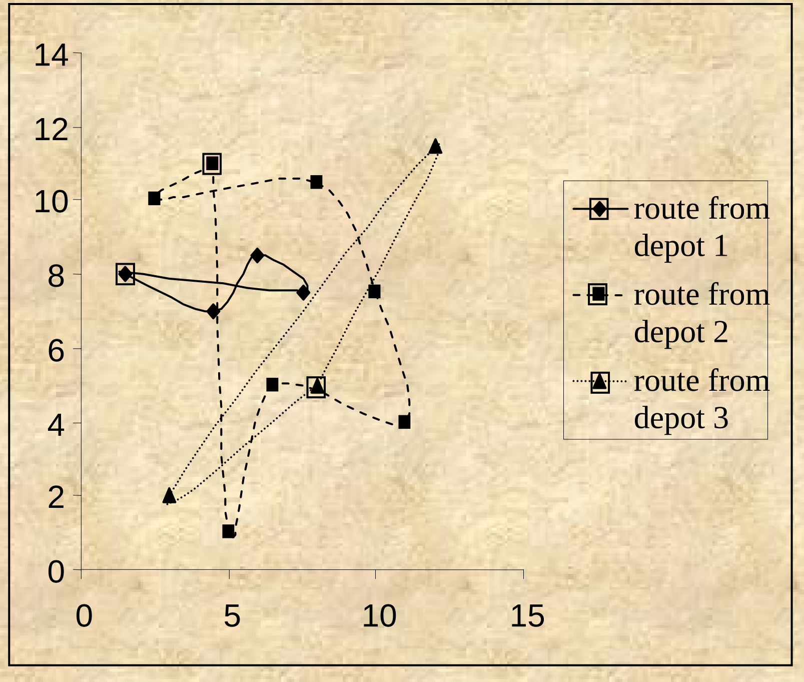
Input Data

- The coordinates of the customers and the depots
- The demand of the customers
- The capacities of the vehicles
- The time windows for different customers
- The probability values for demand for the customers

Generation of initial route set

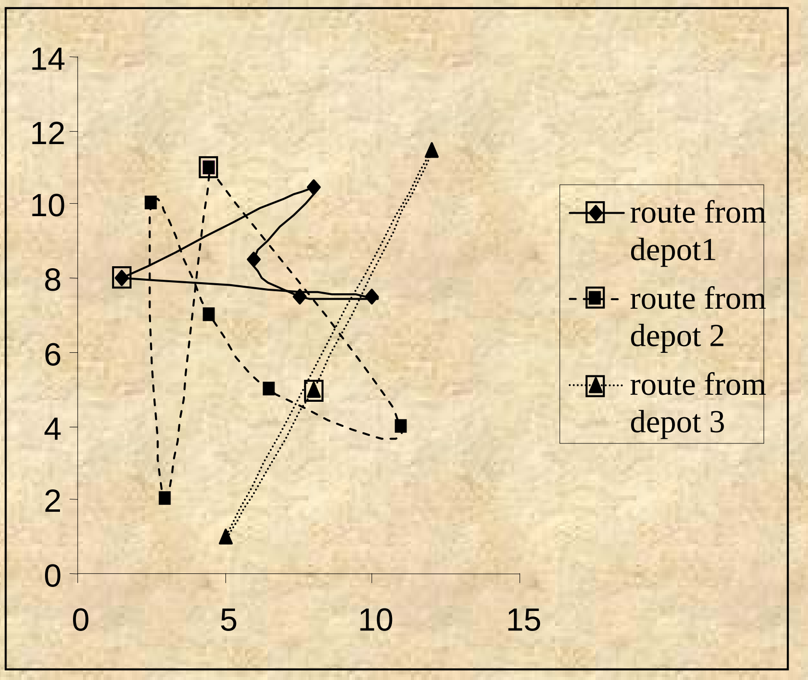


Initial sets of solution



Initial route set 1

(Total travel time: 72.94 units)



Initial route set 2

(Total travel time: 86.56 units)

Evaluation

- The fitness function is given by the following equation

$$TC = \sum\sum d_{ij} \times c_{ij} + C_w \times \sum w_i + C_d \times \sum d_i$$

- where,
- *TC* = Total cost
- d_{ij} = Distance between customer *i* and *j*
- c_{ij} = Cost per unit travel distance
- C_w = Penalty for the early arriving at the customer node *i*
- C_d = Penalty for late arriving at the customer node *i*
- w_i = Waiting time at the customer node *i*
- d_i = Delay time at the customer node *i*

Reproduction

- Tournament selection of size *p*

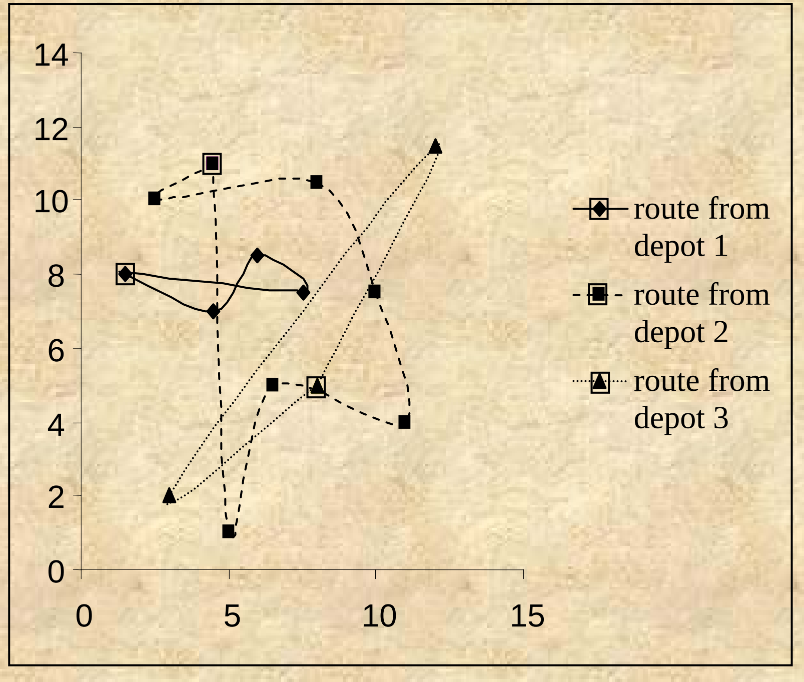
- Mating pool of population size: *mxn*

  where,

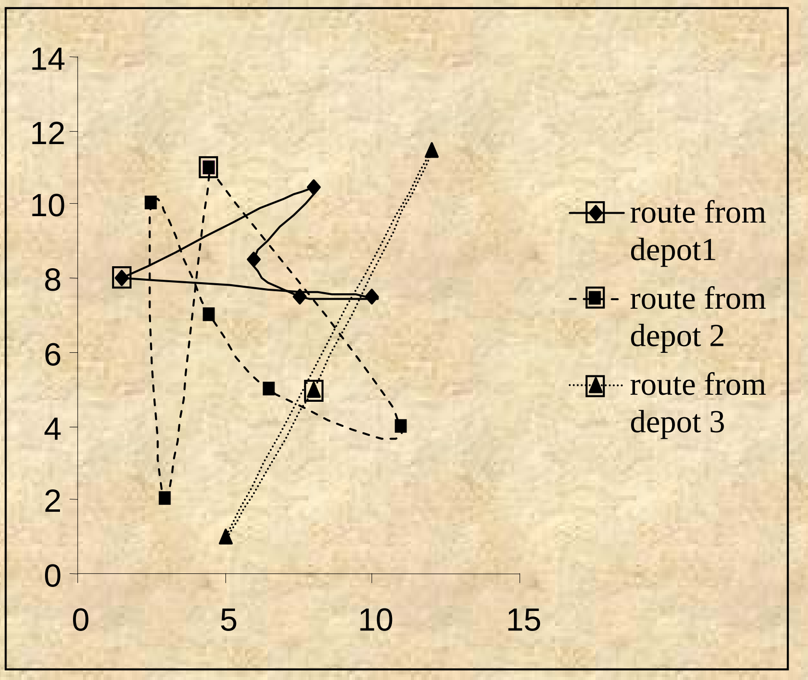
  m: number of better solutions chosen from the initial set of solutions

  n: number of times the m solutions are copied

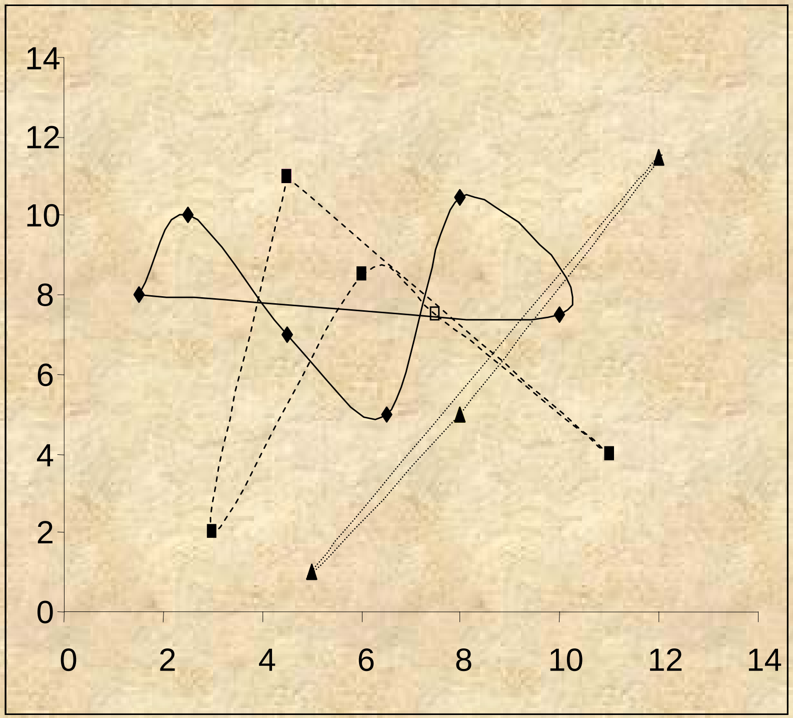
Crossover



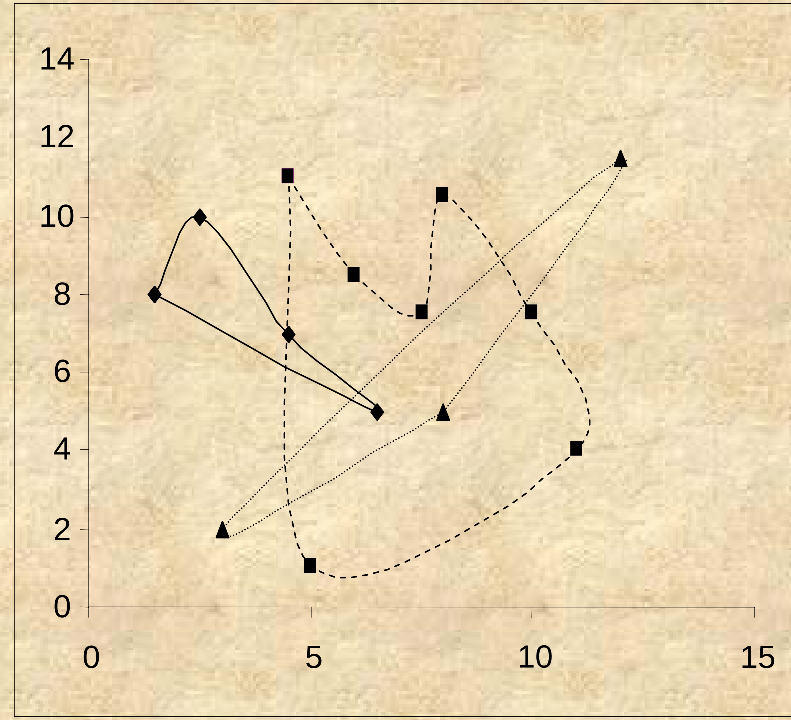
Route set I



Route set II

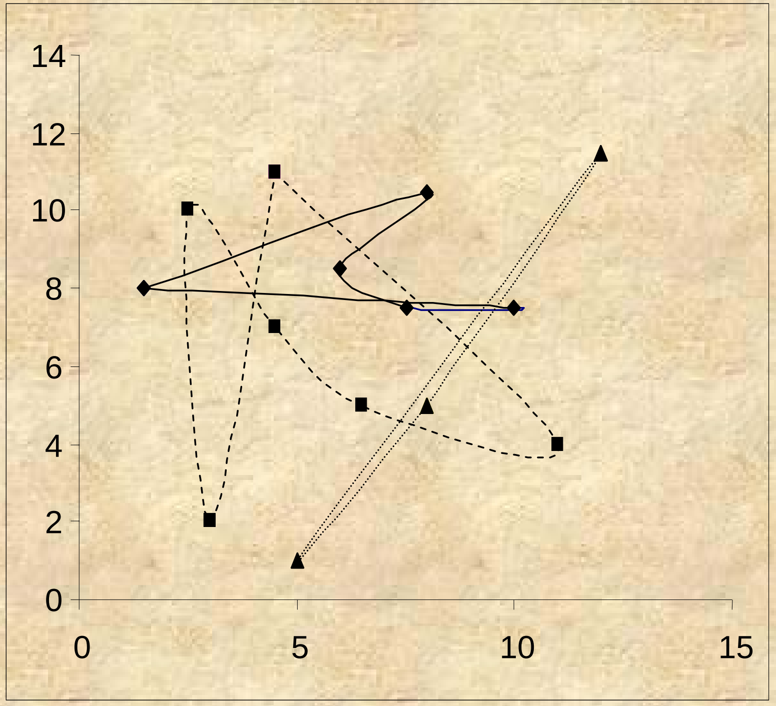


Offspring I by crossover

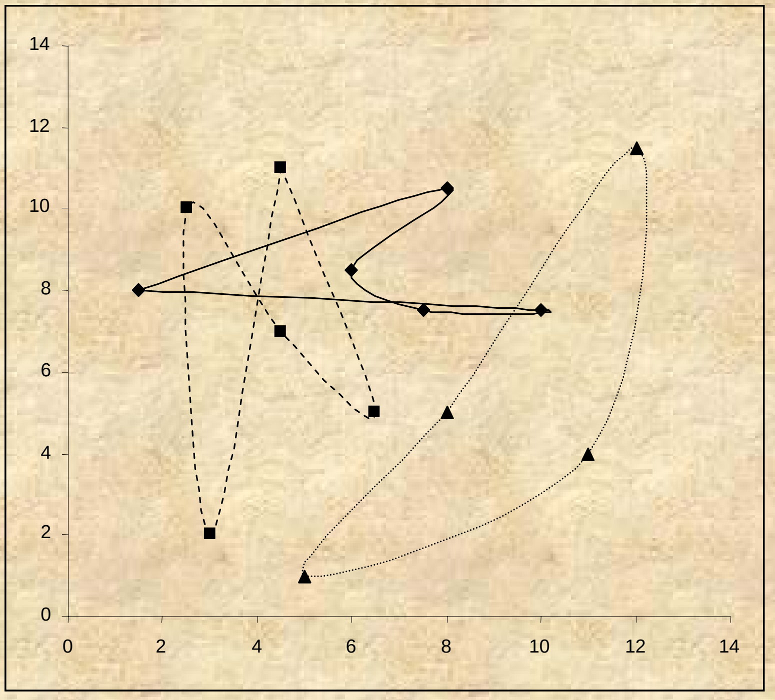


Offspring II by crossover

Mutation

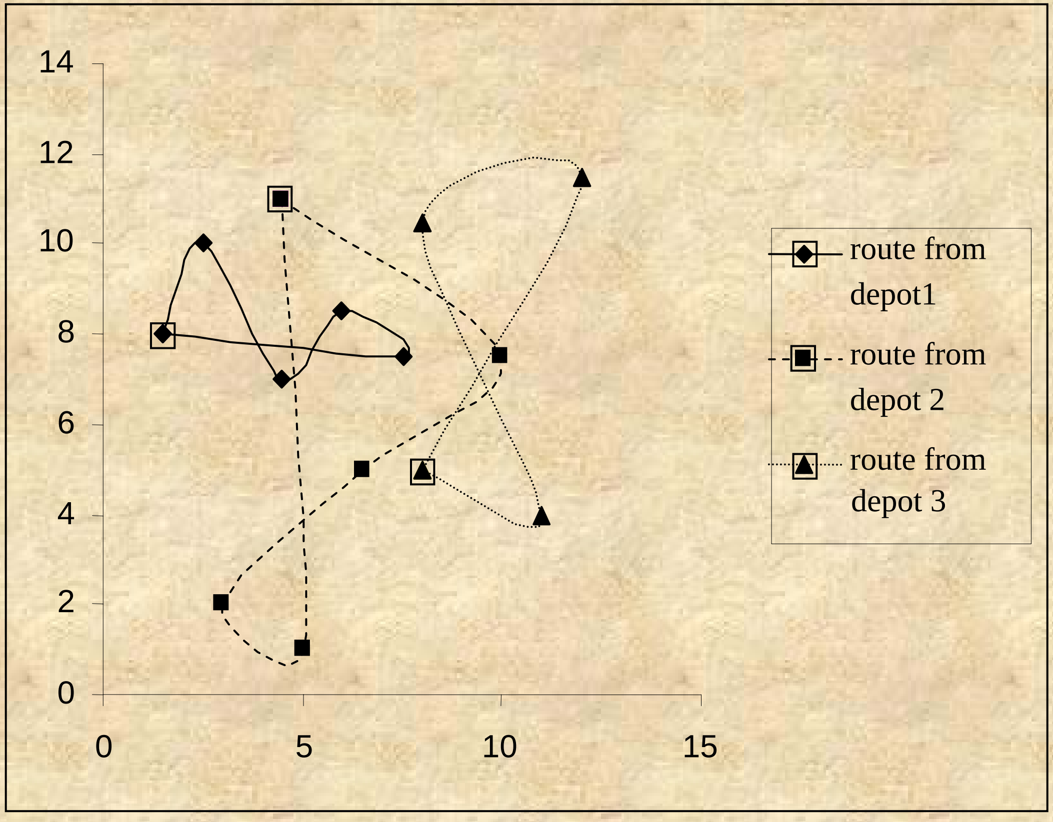


Parent route set



Offspring by mutation

Results



Improved route set obtained using proposed algorithm

(Total travel time: 64.99 units)

Conclusion

- 22% improvement in the travel time is observed using proposed methodology for a problem of 14 nodes

- Future scope of work:
  - further improvement is expected by the sensitivity analysis of the parameters
  - the methodology can be used for larger problems

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Thank you.